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EFFECT OF VISCOSITY ON ENERGY OF HARMONIC OSCILLATOR

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ABSTRACT

due to the wave particle dueling of quantum world one can use energy relations for harmonic oscillator. In this work the kinetic and potential energy, beside the viscous energy for harmonic oscillator and their relation to each other was found. The expression of viscous energy was simplified by relating the classical and quantum expressions according to correspondence principle. This viscous energy expression was added to the classical Hamiltonian energy to find the total medium energy. This total energy beside the wave equation of wave packet was used to find the modified Schrodinger equation.The Schrodinger equation with viscous energy can be used to find the behavior of Hydrogen atoms electrons in bulk mater. The expression reduces to the ordinary ones in the absence of friction. It shows also that the viscous energy is quantized.

KEYWORDS: Law of Stock Force, Laminar and Non Laminar Flow, Harmonic Oscillator.

INTRODUCTION

Quantum mechanics describe the physics of very small particles or the physics of electrons, protons, atom and molecules, and subatomic particles [1]. The beginning of quantum mechanics start by quantization of radiation by Planck postulate in 1900, which consider the energy of standing waves as small tiny quantized particles called photons as proposed by Einstein. The quantization of matter is based on the discovery of De Broglie, the dual nature of atomic particles. Then this is followed by Bohr Theory in 1913 which predicts the existence of atomic electrons energy levels in Hydrogen atom [2].Quantum mechanics strongly affects all fields of science like electronic, spectroscopy, solid state physics, biology as well as medicine, chemistry and laser[3].In spectroscopy, quantum mechanics plays an important role to distinguishes atom by a finger print of spectrum. It also determine the shape, size, composition, radiation and decay of nuclear particles[4]. The term viscosity is commonly used for the description of fluid flow to characterize the degree of internal friction in the fluid. This internal friction, or viscous force, is associated with the resistance that two adjacent layers of fluid have when they move relative to each other. Viscosity causes part of the kinetic energy of a fluid to be converted to internal energy. This mechanism is similar to the one by which an object sliding on a rough horizontal surface loses kinetic energy [5].The flow of particle in viscous media is either laminar or non laminar flow according to satisfy critical value called Renold's number. This is done in section (2), sections (3) and (4) are concerned with discussion and conclusion.The aim of present study is modified energy of harmonic oscillator using Stock's force to derive an expression for energy lost by the particles due to viscosity. This expression of energy is used to derive quantum equation that account for the effect of viscosity. The discussion and conclusion are respectively in section (3) and (4).

WORK DONE BY STOCK FORCE IN OSCILLATION SYSTEM

The energy loss due to harmonic oscillator vibration in viscous media can be obtained in terms of Stock force act on it as

$$E_{vis} = \int F_{Stock}\, dx = \int \beta\, v dx \qquad (1)$$

The value of β for laminar and non laminar flow are defined by Eqn. (2) and Eqn. (3) respectively as

$$\beta_6 = 6\pi\eta_h\gamma_h \qquad (2)$$

$$\beta_7 = \frac{1}{2}\frac{\eta_h\, N_{gh}}{d_h}C_{Dh}A \qquad (3)$$

The position of oscillation system is given by

$$x = x_0 e^{i\omega t} \qquad (4)$$

But the linear velocity can be obtain from first derivative of position as

$$v = \frac{dx}{dt} = i\omega x$$

Or

$$v = i\omega x \qquad b \qquad (5)$$

Sub Eqn. (5) in Eqn. (1) one get

$$E_{vis} = i\omega\beta\int x\,dx = \frac{i\omega\beta}{2}x^2$$

$$E_{vis} = \frac{i\omega\beta}{2}x^2 \qquad (6)$$

And the energy loss due to viscose media operator as

$$\hat{E}_{vis} = \frac{i\omega\beta}{2}\hat{x}^2 \qquad (7)$$

The eigen value of energy loss operator is

$$\frac{i\omega\beta}{2}\hat{x}^2\psi(x) = \frac{i\omega\beta}{2}x^2\psi(x) \qquad (8)$$

THE EFFECT OF VISCOSITY ON OSCILLATION SYSTEM

The equation of motion of harmonic oscillator can be expressed as

$$x(t) = x_0 e^{i\omega t} \qquad (9)$$

Velocity of harmonic oscillator is the rate of change of $x(t)$ thus

$$v = i\omega x(t) \qquad (10)$$

The kinetic energy is given by

$$T = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2x^2 \qquad (11)$$

The potential energy takes the form

$$V = -\int F\,dx = k\int x\,dx = \frac{1}{2}m\omega^2x^2 \qquad (12)$$

Thus for harmonic oscillator, potential and kinetic energy are equal as a result

$$E = T + V = 2T = m\omega^2x^2 \qquad (13)$$

Eqn. (6) can be rewritten as

$$E_{vis} = \frac{i\beta m\omega^2}{2\omega m}x^2 = \frac{i\beta E}{2\omega m} \qquad (14)$$

But from quantum Planck harmonic quantum relation

$$E = \hbar\omega$$

Therefore

$$E_{vis} = \frac{i\beta\hbar\, E}{2\hbar\,\omega m} = \frac{i\beta\hbar\, E}{2Em} = \frac{i\beta\hbar}{2m} \qquad (15)$$

The Hamiltonian of harmonic oscillator can be written as

$$H = \frac{P^2}{2m} + V + \frac{i\beta\hbar}{2m} \qquad (16)$$

The eigen equation of harmonic oscillator is thus given by

$$\hat{H}\psi(x) = \left[\frac{\hat{P}^2}{2m} + \hat{V} + \frac{i\beta\hbar}{2m}\right]\psi(x)$$

$$i\hbar\frac{\partial\psi(x)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + \left[V + \frac{i\beta\hbar}{2m}\right]\psi(x) \qquad (17)$$

Harmonic oscillator is independent of time, since Eqn. (17) convenient to be written as independent time Schrodinger equation in the form

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}\left[E - \frac{1}{2}m\omega^2x^2 + \frac{i\beta\hbar}{2m}\right]\psi(x) = 0 \qquad (18)$$

Transform this equation into a variable ξ defined by

$$\xi = \acute{\alpha}x \qquad (19)$$

The parameter $\acute{\alpha}$ will be chosen in such a way that the new equation looks simple.

$$\frac{d\psi}{dx} = \frac{d\psi}{d\xi}\frac{d\xi}{dx} = \acute{\alpha}\frac{d\psi}{d\xi}$$

$$\frac{d^2\psi(x)}{dx^2} = \acute{\alpha}\frac{d}{dx}\left(\frac{d\psi}{d\xi}\right) = \acute{\alpha}\frac{d}{d\xi}\left(\frac{d\psi}{d\xi}\right)\frac{d\xi}{dx} = \acute{\alpha}^2\frac{d^2\psi}{d\xi^2}$$

In terms of new variable Eqn. (18) becomes

$$\frac{d^2\psi}{d\xi^2} + \left[\frac{2mE}{h^2\acute{\alpha}^2} - \frac{m^2\omega^2}{h^2\acute{\alpha}^4}\xi^2 + \frac{i\beta}{2\hbar\acute{\alpha}^4}\xi^2\right]\psi = 0 \qquad (20)$$

Assume $\acute{\alpha}^4$ define as

$$\acute{\alpha}^4 = \frac{m^2\omega^2 - i\pi\beta h}{h^2} \qquad (21)$$

Equation (20) now becomes

$$\frac{d^2\psi}{d\xi^2} + \left[\frac{2mE}{h^2\acute{\alpha}^2} - \xi^2\right]\psi = 0 \qquad (22)$$

Introducing the dimensionless parameter $\mathcal{B}$ defined by

$$\mathcal{B} = \frac{2mE}{h^2\acute{\alpha}^2} \qquad (23)$$

Eqn. (22) becomes

$$\frac{d^2\psi}{d\xi^2} + [\mathcal{B} - \xi^2]\psi = 0 \qquad (24)$$

The steps of solution is known until obtain value of $\mathcal{B}$ given by

$$\mathcal{B} = \frac{2mE}{h^2\acute{\alpha}^2} = 2n + 1 \qquad (25)$$

The energy eigen value is

$$E_n = \left(n + \frac{1}{2}\right)\frac{h^2\acute{\alpha}^2}{m} = \left(n + \frac{1}{2}\right)\frac{h^2}{m}\sqrt{\frac{m^2\omega^2 - i\pi\beta h}{h^2}}$$

Or

$$E_n = \left(n + \frac{1}{2}\right)h\omega\sqrt{1 - \frac{i\pi\beta}{m^2\omega^2 h}} \qquad (26)$$

Or

$$\beta = \frac{m^2}{i\pi\beta h\left(n + \frac{1}{2}\right)}\left[\frac{\left[\left(n + \frac{1}{2}\right)h\omega\right]^2 - E_n{}^2}{\left[\left(n + \frac{1}{2}\right)h\omega\right]^2}\right] \qquad (27)$$

For laminar flow substitute the value of β from Eqn. (2) in Eqn. (27) to obtain viscosity coefficient as

$$\eta_h = \frac{m^2}{i6\pi^2\gamma_h h\left(n + \frac{1}{2}\right)}\left[\frac{\left[\left(n + \frac{1}{2}\right)h\omega\right]^2 - E_n{}^2}{\left[\left(n + \frac{1}{2}\right)h\omega\right]^2}\right] \qquad (28)$$

But for non laminar flow viscosity coefficient can be obtained by substitute Eqn. (3) in Eqn. (27) as

$$\eta_h = \frac{2d_h m^2}{i\pi h N_{gh} C_{Dh} A\left(n+\frac{1}{2}\right)}\left[\frac{\left[\left(n+\frac{1}{2}\right)h\omega\right]^2 - E_n}{\left[\left(n+\frac{1}{2}\right)h\omega\right]^2}\right]^2 \qquad (29)$$

DISCUSSION

According to equation (6) and (7) the energy lost due to viscosity is dependent on x^2, ω as well as viscosity coefficientβ. This is obvious, since increasing x and ω besidesβ, increases losses. Using classical and quantum expressions of oscillator [see Eqn. (3), (14) and (15)], the viscous energy is constant which depends on β as well as h and m. The Hamiltonian and Schrodinger equation are given by Eqns. (16-18). Equation (17) reduces to ordinary Schrodinger equation in the absence of friction. The energy in equation (26) consists of an additional term consisting of viscosity coefficientβ. This expression reduces to that of ordinary harmonic oscillator in the absence of friction. Equation (28) indicates that the friction is quantized.

CONCLUSION

The quantum viscose model reduces to Schrodinger equation in the absence of friction. For harmonic oscillator it shows that the energy is quantized.

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